(II) Angle θ between the two lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$ is

$$\cos\theta = \left| \frac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}|\,|\vec{b_2}|} \right|$$

In Cartesian form

If a_1, b_1, c_1 and a_2, b_2, c_2 are the drs of two lines then angle θ is given by

$$\cos\theta = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

Non-coplanar lines / skew lines : Lines which are neither intersecting nor parallel

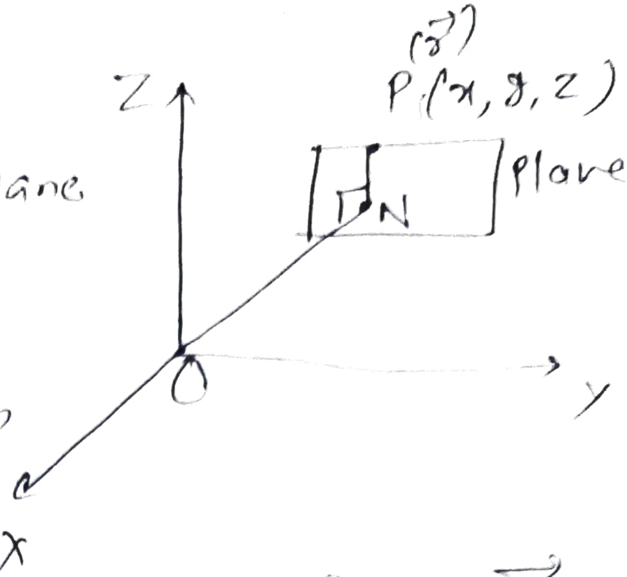
Distanced between skew lines $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

or S.D. between two lines $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$ is

$$= \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2 + (a_1 b_2 - a_2 b_1)^2}}$$

Plane



Let the distance of plane from origin $= d$.

Let $\vec{ON}$ is normal from origin to plane and

$\hat{n}$ = unit normal vector along $\vec{ON}$. $\Rightarrow \vec{ON} = d\hat{n}$

Let $P(x, y, z)$ be a point with position vector $\vec{r}$ on the plane

$\Rightarrow$ $\vec{NP} \perp \vec{ON}$

vector form of the eqn is $\boxed{\vec{r} \cdot \hat{n} = d}$

Cartesian form of the plane is $\boxed{ax + by + cz + d = 0}$

or $\boxed{lx + my + nz = d}$

where l, m, n are d.cs of the plane.

* i.e. Equation of a plane (where $\vec{n}$ is unit normal vector) then

$$\vec{r} \cdot \hat{n} = d \;(= \text{distance from origin to the plane})$$

* Eqn of a plane $\perp$r to a vector $\vec{N}$ and passing through a point (with p.v. $\vec{a}$) is

$$\boxed{(\vec{r} - \vec{a}) \cdot \vec{N} = 0}$$

Eqn of a plane $\perp$r to a line with drs A, B and C through a point $A(x_1, y_1, z_1)$ is

$$\boxed{A(x - x_1) + B(y - y_1) + C(z - z_1) = 0}$$

* Eqn of a plane through 3 points $\vec{a}$, $\vec{b}$, and $\vec{c}$ is

$$(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$$

* cartesian eqn of a plane through 3 points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$